



Syllabus in M. Sc. Mathematics gets changed both in structure as well as in content beginning from June-2022 progressively. Since new changed syllabus has structural change, it is not possible to carry forward credits earned in old course to new course or the other way.

Only three trials (i.e. 1 + 2) for old course exam will be given. Old course students will have to clear M. Sc. Mathematics taking benefit of these trials. If they fail to do so, they will have to take fresh admission and earn all new credits from the new course.

# M. Sc. Mathematics Syllabus For Credit Based Semester System Revised (2022)

Semester	Course Code	Course Name	Hours per Week	Course Credits
I	MAT401	Mathematical Statistics	4	4
	MAT402	Measure & Integration	4	4
	MAT403	Complex Analysis – I	4	4
	MAT404	Ordinary differential Equations	4	4
	MAT405PR	Practical – I (Mathematical Statistics & Measure & Integration)	6	4
	MAT406PR	Practical – II (Complex Analysis & ODE)	6	4
		Total	28	24
II	MAT407	Metric Spaces	4	4
	MAT408	Real Analysis	4	4
	MAT409	Complex Analysis – II	4	4
	MAT410	Partial Differential Equations	4	4
	MAT411PR	Practical – I (Metric Spaces & Real Analysis)	6	4
	MAT412PR	Practical – II (Complex Analysis & PDE)	6	4
		Total	28	24
111	MAT501	Functional Analysis – I	4	4
	MAT502	Algebra – I	4	4
	MAT503EA	Advanced Calculus	4	Δ
	MAT503EB	Financial Mathematics	4	4
	MAT504EA	Mathematical Programming	4	4
	MAT504EB	Computational Mathematics		
	MAT505PR	Practical – I (Functional Analysis-I & Algebra-I)	6	4
	MAT506PR	Practical – II (Based on MAT503 & MAT504)	6	4
		Total	28	24
IV	MAT507	Differential Geometry	4	4
	MAT508	Algebra – II	4	4
	MAT509EA	Number Theory	4	4
		Mathematical Methods		
	MAT510EA	Quantitative Techniques	4	4
	MAT511PR	Problem Solving	6	<u> </u>
	ΜΔΤ51202	Project	6	
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		l		50

## **MAT401: Mathematical Statistics**

#### **Objectives and Outcome:**

Statistical methods used in practice are based on the foundations of statistical theory. One branch of this theory uses the tools of probability to establish important distributional results that are used throughout statistics. Another major branch of statistical theory is statistical inference. This basic course toward the first branch is concerned with the fundamental theory of probability, random variables, Expectation, Distributions. Students will be familiar with many common distributions, continuous or discrete, univariate or multivariate, which provides rich families for modeling real data.

- UNIT I: An introduction to Probability: Review of sets, Experiments and Sample spaces, Events, Probability Definition and Assignment, Finite Sample spaces and enumeration, Conditional Probability, Partitions, Total Probability and Bayes' Theorem.
  One-Dimensional Random Variables: The Distribution Function, Discrete Random Variables, Continuous Random Variables, Characteristics of Distributions, Chebyshev's Inequality.
- UNIT II: Functions of One Random Variable and Expectation: Equivalent Events, Functions of a Discrete Random Variable, Continuous functions of a continuous random variable, Expectation, Approximations to E[H(x)] and V[H(X)], Moment Generating Function. Joint Probability Distributions: Joint Distribution for Two-Dimensional Random variables, Marginal Distributions, Conditional Distributions, Conditional Expectation.
- **UNIT III:** Discrete Distribution: Bernoulli Trials and the Bernoulli Distribution, Binomial Distribution, Pascal Distribution, Hypergeometric Distribution, Poisson Distribution. Continuous Distributions: The Uniform Distribution, Exponential Distribution, Weibull Distribution.
- **UNIT IV:** Gamma Distribution, Beta Distribution, The Normal Distribution, The reproductive property of normal distribution, Central Limit Theorem, The Normal Approximation to the Binomial Distribution, The Lognormal Distribution.

#### Textbook:

This course is roughly covered by Chapters 1, 2, 3, 5, 6, 7 of Probability and Statistics in Engineering 4th edition – Hines, Montgomery, Goldsman and Borror, published by Wiley (India Edition).

#### References

- 1. A first course in probability by Sheldon Ross, Pearson Education.
- 2. Probability and Statistics for Engineers by Richard Johnson, Prentice Hall of India Publication.
- 3. Fundamentals of Mathematical Statistics by Kapoor & Gupta, Sultan Chand and Sons.

## MAT402 – Measure and Integration

#### **Objectives and Outcome:**

The initial objective of the course is to introduce the concept of Lebesgue measure for bounded subsets of R. This concept of Lebesgue measure is later used in developing the theory of (Lebesgue) integration which gives stronger (and better) results as compared to the theory of Riemann integration.

- **Unit I:** The structure of open sets in R, Length of open sets and closed sets, Inner and outer measure of bounded sets, Measurable sets and some of its properties.
- **Unit II:** Further properties of measurable sets, Non-measurable sets, Definition and the properties of Measurable functions.
- **Unit III:** A quick review of the definition of Riemann integral, Lebesgue integral for bounded functions and its comparison with Riemann integral, properties of Lebesgue integral for bounded functions.
- **Unit IV:** The Lebesgue integral of non-negative and unbounded functions, its properties, Lebesgue dominated convergence theorem, Fatou's Lemma and its consequences like Monotone convergence theorem and the countable additivity of the Lebesgue integral, A very brief introduction to Lebesgue integral on  $(-\infty,\infty)$  and in plane.

#### Text book:

The course is based on the book "Methods of Real Analysis" by Richard Goldberg, Oxford & IBH Publishing Company, 1964.

- 1. "Theory of Functions of a real variable" Volume-I by I. P. Natanson, Frederic Ungar Publishing Co., New York 1964.
- 2. "Real Analysis" by H. L. Royden (3rdedition), Pearson Prentice Hall (2007).
- 3. "Measure and Integration" by I. K. Rana, Narosa Publishing House (1997).

## MAT403 – Complex Analysis-I

#### **Objectives and outcome:**

The main aim is to make students familiar with complex numbers, their properties and the study of functions of a complex variable. It is expected that on successful completion of this course the students will be able to handle complex numbers and complex valued functions of complex variables with Mathematical Maturity.

- Unit I: Basic Definitions and notations, Algebraic properties, Polar coordinates and Euler's formula, Products and Quotients in exponential form. Roots of complex Numbers. Continuous complex functions. Limits Involving Point at Infinity.
- **Unit II:** Differentiable complex functions, Cauchy Riemann equations. Harmonic functions of two variables, Reflection principle.
- **Unit III**: Elementary functions, Contours, Contour integrals, Anti-derivatives.
- **Unit IV:** Cauchy-Goursat theorem, Simply connected domain, Multiply connected domains. Cauchy's integral formula and its Extension. Liouville's theorem, Fundamental theorem of Algebra, Maximum moduli principle of functions.

#### Text book:

The course is roughly covered by "Complex variables and Applications" (8th edition), J. W. Brown and R. V. Churchill, McGraw Hills. International Edition 2009. ISBN: 978-007-126328-3. OR MHID: 007-126328-4.

- 1. "Introduction to Functions of Complex Variable" C. J. Hamilton, Marcel Dekker Inc. New York.
- 2. "Complex Analysis" I. Stewart and David Tall, Cambridge University Press.
- 3. "Complex Analysis" J. C. Duncan, John Wiley & Sons, London.
- 4. "Complex Analysis" Lars Ahlfors, McGraw Hills. Indian Edition.

## **MAT404 – Ordinary Differential Equations**

#### **Objectives and outcome:**

The objective of this course is to continue the study of ordinary differential equations begun in B. Sc., with an emphasis on second degree equations which occur in applications.

- **Unit I:** Review of second order linear equations. Series solutions of first order equations. Second order linear equations: ordinary points.
- **Unit II**: Second order linear equations: regular singular points; Gauss's hypergeometric equation; the point at infinity.
- **Unit III:** Hermite polynomials. Chebyshev polynomials and the minimax property. Legendre polynomials; properties of Legendre polynomials.
- **Unit IV:** Bessel functions; properties of Bessel functions; Bessel's integral formula. Existence and uniqueness of solutions: the method of successive approximations; Picard's theorem; systems of equations.

#### Text book:

"Differential Equations with Applications and Historical Notes" (2nd Edition) - G. F. Simmons, Tata McGraw-Hill Publishing Co. Ltd., 2010. Chapter 5 (Omit Appendices C and E), Chapter 8 (Omit Appendices A and B), Chapter 13.

- 1. "Introduction to Ordinary Differential Equations"
  - A. L. Rabenstein, Academic Press.
- "Advanced Engineering Mathematics" (10th Edition)
  Erwin Kreyszig, Wiley-India, 2011.

## MAT405PR – Practical-I

#### Practicals Based on Mathematical Statistics

- 1) Probability and Bayes' Theorem
- 2) The Distribution Function Discrete and Continuous Random Variables
- 3) Expectation and Moments
- 4) Bernoulli Trials and the Bernoulli Distribution, Binomail Distribution
- 5) Hypergeometric Distribution, Poisson Distribution
- 6) The Uniform Distribution, Exponential Distribution,
- 7) Gamma Distribution, Beta Distribution, Weibull Distribution
- 8) Normal Distribution

#### Textbook:

This course is roughly covered by Chapters 1, 2, 3, 5, 6, 7 of Probability and Statistics in Engineering 4<sup>th</sup> edition – Hines, Montgomery, Goldsman and Borror, published by Wiley (India Edition)

#### **References:**

- 1. A first course in probability by Sheldon Ross, Pearson Education
- 2. Probability and Statistics for Engineers by Richard Johnson, Prentice Hall of India Publication
- 3. Fundamentals of Mathematical Statistics by Kapoor & Gupta, Sultan Chand and sons

### Practicals based on Measure and Integration

- 1) The least upper found (sup) and greatest lower bund (inf) of a subset of R.
- 2) Pointwise and uniform convergence of a sequence of functions.
- 3) Outer measure and inner measure of a subset of [a,b].
- 4) Measurable sets and Measurable functions.
- 5) Lebesgue integral for bounded measurable functions.
- 6) Lebesgue integral for unbounded measurable functions.
- 7) Fatou's Lemma and its consequence.
- 8) Lebesgue integral on  $(-\infty, \infty)$  and in plane.

- 1. Methods of Real Analysis by Richard Goldberg, Oxford & IBH Publishing Company, 1964.
- 2. Theory of Functions of a real variable Volume-I by I. P. Natanson, Frederic Ungar Publishing Co., New York 1964.
- 3. Measure and Integration by Satish Shirali and H L Vasudeva, Springer, 2019.
- 4. Real Analysis by H. L. Royden and P M Fitzpatrick, Pearson Education Asia Limited and China Press, Fourth Edition, 2010.
- 5. Measure and Integration by I. K. Rana, Narosa Publishing House (1997).

## MAT406PR – Practical II

#### Practicals Based on MAT403: Complex Analysis-I and MAT404: Ordinary Differential Equations

#### **Objectives and outcome:**

The objective here is to acquaint students to develop and consolidate their problem solving skills with the problems related to the notions that they have familiarize themselves in Complex Analysis-I and Ordinary Differential Equation courses. It is expected that on successful completion of this course the students will be able to handle problems related to the topics mentioned in the title with enhanced Mathematical Maturity.

#### Practicals Based on MAT403: Complex Analysis-I

- 1) Problems Based on Finding Argument, Sketching the sets, Finding roots and sketching them. Also, problems on finding limit involving infinity.
- 2) Problems Based on C-R equations and the sufficient condition for differentiability. Also, in Polar Form.
- Rough sketches of images of Horizontal and Vertical lines under the exponential map e<sup>z</sup>.
  Examples on Finding Harmonic Conjugate. Examples on Reflection Principle.
- 4) Examples Based on e<sup>z</sup>, Log z, z<sup>c</sup>.
- 5) Problems Based on trigonometric, Hyperbolic functions and their inverses.
- 6) Examples on Contour Integrals and their upper bound. Examples Based on deformation of paths.
- 7) Examples Based on Cauchy Integral Formula and its Extension.
- 8) Examples Based on Maximum Modulus Principle. And Miscellaneous Problems.

#### Practicals Based on MAT404: Ordinary Differential Equations

- 1. First order linear equations, second order linear homogeneous equations with constant coefficient, the use of a known solution to find another solution.
- 2. Second order linear nonhomogeneous equations (Method of undetermined coefficients and Method of variation of parameters).
- 3. Second order linear equations (Power series method and Frobenius method).
- 4. Second order linear equations (Power series method and Frobenius method).
- 5. Hermite polynomials, Chebyshev polynomials and Legendre polynomials.
- 6. Bessel functions.
- 7. Bessel functions.
- 8. The Picard's method of successive approximations.

#### **Reference Books:**

- 1. "Differential Equations with Applications and Historical Notes" (2<sup>nd</sup> Edition) G. F. Simmons, Tata McGraw-Hill Publishing Co. Ltd., 2010
- 2. "Introduction to Ordinary Differential Equations" A. L. Rabenstein, Academic Press.
- 3. "Advanced Engineering Mathematics" (10<sup>th</sup> Edition) Erwin Kreyszig, John Wiley and Sons Inc., 2011.

**Note:**Whenever possible it is expected that teachers will incorporate and encourage the use of packages like Mathematica/MATLAB/Maple/etc. in solving the problems in practical sessions.

## MAT407 – Metric Spaces

#### **Objectives and outcome:**

The objective of this course is to give a very streamlined development of a course in metric space topology emphasizing only the most useful concepts, concrete spaces and geometric ideas, which are useful in almost all courses of mathematics.

- **Unit I:** Metric and Metric Spaces, Metric from an inner product and a norm, Open balls and Open sets, Equivalent metrics, Interior of a set, Subspace topology, Closed sets. Omit proofs of Cauchy-Schwarz, Young's, Hölder's and Minkowski Inequalities.
- Unit II: Convergence of a sequence, Limit and Cluster points, Bolzano Weierstrass Theorem, Cauchy sequences and Completeness, Bounded sets, Dense sets, Basis, Boundary of a set.
- **Unit III:** Continuous functions, Equivalent definition of continuity, Distance between two sets, Urysohn's Lemma for metric space, Gluing Lemma, Topological property, Uniform continuity, Limit of a function, Open and closed maps.
- **Unit IV:** Compact spaces and their properties, Continuous functions on Compact spaces, Characterization of compact metric spaces. Connected spaces, Product of two connected spaces, Path connected spaces.

#### Text book:

Chapter 1 to 5 from "Topology of Metric Spaces" (2nd Edition) - S. Kumaresan, Narosa Publishing House, 2011.

- 1. "Introduction to Real Analysis"
  - R. G. Bartle and D. R. Sherbert (3rd edition), John Wiley and Sons (ASIA), 2000
- 2. "Principles of Mathematical Analysis"
  - Walter Rudin (3rd edition), International Student Edition, McGraw-Hill, 1985.
- 3. "Metric Spaces"
  - Satish Shirali and Harkrishan L. Vasudeva, Springer International Edition, 2006.
- 4. "Metric Spaces"
  - Micheál Ó Searcóid, Springer International Edition, 2007.

## MAT408 – Real Analysis

#### **Objectives and Outcome:**

The main objective of the course is to study the differential properties of functions of finite variation and absolutely continuous functions and characterize the absolutely continuous functions in terms of the indefinite integral of Lebesgue integrable functions.

A part of the course is also devoted to the study of the structure of measurable functions and study of Fourier series of  $L_1$  and  $L_2$  functions.

- **Unit I:** Convergence in measure and the related important results, Approximations of measurable functions by bounded measurable functions and continuous functions, Weierstrass approximation theorems.
- **Unit II:** Square-summable functions, Schwarz and Minkowski's inequality, Completeness of  $L_2[a, b]$ , Dense subsets of  $L_2[a, b]$ ,  $L_p[a, b]$  as a generalization of  $L_2[a, b]$ , a quick introduction of sequence spaces  $l_2$  and  $l_p$ .
- **Unit III:** Monotonic function and its differentiability, functions of finite (bounded) variation on [a,b] and its properties, Absolutely continuous functions on [a,b], differential properties of absolutely continuous function.
- **Unit IV:** The indefinite Lebesgue integral and the fundamental theorem of calculus, Definition of Fourier series and convergence problem, (C,1) summability of Fourier series, the L<sub>2</sub> theory of Fourier series, Convergence of Fourier series.

#### Text books:

The course is based on the following books:

- 1. "Theory of Functions of a real variable" Volume-I by I. P. Natanson, Frederic Ungar Publishing Co., New York 1964.
- 2. "Methods of Real Analysis" by Richard Goldberg, Oxford & IBH Publishing Company, 1964.

- 1. "Real Analysis" by H. L. Royden (3<sup>rd</sup> edition), Pearson Prentice Hall (2007).
- 2. "Measure and Integration" by I. K. Rana, Narosa Publishing House (1997).

## MAT409 – Complex Analysis-II

#### **Objectives and outcome:**

The main aim is to make students familiar with Contour Integrals and Residue Theory for finding Contour Integrals. It is expected that on successful completion of this course the students will be able to handle Contour Integrals and its applications with Mathematical Maturity.

- **Unit I:** Convergence of Taylor series, Laurent series and Uniqueness, Convergence of sequences and series, Uniform and absolute convergence of power series. Multiplication and Division of Power Series.
- **Unit II:** Residue, Cauchy's Residue theorem, Residue at Infinity. Types of isolated singular points, Residues at poles, Zeros and poles of order m, Behavior of f near removable and essential singular points.
- **Unit III:** Evaluation of improper integrals from Fourier Analysis using Residues. Jordan's Lemma. Indented Paths. Indentation around a Branch cut. Integration along a Branch Cut. Definite integrals involving Sines and Cosines using Residues.
- **Unit IV:** Argument Principle, Rouché's Theorem and Möbius Transformations (Bi-linear transformation).

#### Text book:

The course is roughly covered by "Complex variables and Applications" (8th edition) J. W. Brown and R. V. Churchill, McGraw Hills. International Edition 2009. ISBN: 978-007-126328-3. OR MHID: 007-126328-4.

- 1. "Introduction to Functions of Complex Variable" C. J. Hamilton, Marcel Dekker Inc. New York.
- 2. "Complex Analysis" I. Stewart and David Tall, Cambridge University Press.
- 3. "Complex Analysis" J. C. Duncan, John Wiley & Sons, London.
- 4. "Complex Analysis" Lars Ahlfors, McGraw Hills. Indian Edition

## **MAT410 – Partial Differential Equations**

#### **Objectives and outcome:**

The objective of this course is to introduce partial differential equations, particularly the second order equations of mathematical physics.

- **Unit I:** Review of curves and surfaces; genesis of first order PDE; classification of integrals; linear equations of the first order; Pfaffian differential equations; compatible systems of first order PDE, Charpit's method.
- **Unit II:** Jacobi's method; integral surface through a given curve; quasi-linear equations (characteristic curves and the initial value problem), Non-linear first order PDE (characteristic curves and the initial value problem).
- **Unit III:** Genesis of second order PDE; classification of second order PDE, Introduction to the initial and boundary value problems, One dimensional wave equation; vibrations of an infinite string; Vibrations of a semi-infinite string, Vibrations of a string of finite-length, Heat conduction problem: infinite rod; finite rod.
- **Unit IV:** Duhamel's principle, Laplace's equation: boundary value problems; maximum and minimum principles; the Dirichlet problem for a circle, for the upper half plane, for a rectangle, Neumann's problem for the upper half plane and for a circle, Harnack's theorem; Green's function, Families of equipotential surfaces.

#### Text book:

"An Elementary Course in Partial Differential Equations" (2nd Edition) – T. Amaranath, Narosa Publishing House Pvt. Ltd., 2005.

- 1. Elements of Partial Differential Equations: Ian Sneddon, Mc-Graw-Hill International Editions.
- 2. Ordinary and Partial Differential Equations: Theory and Application Nita H Shah, PHI Learning Pvt Ltd.
- 3. Partial Differential Equations: Methods, Applications and Theories Harumi Hattori, World Scientific.
- 4. Elements of Partial Differential Equations (2nd Edition) Pavel Drabek, Gabriela Holubova, DE GRUYTER.
- 5. Linear Partial Differential Equation for Scientists and Engineers: by Tyn Myint-U and Lokenath Debnath, Fourth Edition, Birkauser Publications.
- 6. Methods of Mathematical Physics Vol.2, R. Courant and D. Hilbert, Wiley Eastern Pvt. Ltd., 1975.

## MAT411PR – Practical I

#### Practicals Based on Metric Spaces:

- 1) Examples of Metric spaces.
- 2) Open sets and closed sets, interior of a set.
- 3) Convergent sequences, limit and cluster points, Closure of a set.
- 4) Cauchy sequences and completeness, bounded sets, dense sets, boundary of a set.
- 5) Continuous functions.
- 6) Homeomorphism, uniform continuity.
- 7) Compact spaces and their properties.
- 8) Connected spaces and their properties.

#### **Reference Books:**

- 1. Topology of Metric Spaces S. Kumaresan, Narosa Publishing House, 2011
- 2. Principles of Mathematical Analysis Walter Rudin (3rd edition), International Student Edition, McGraw-Hill, 1985.
- 3. Metric Spaces Satish Shirali and Harkrishan L. Vasudeva, Springer International Edition, 2006.
- 4. Metric Spaces Micheál Ó Searcóid, Springer International Edition, 2007.

#### **Practicals Based on Real Analysis**

- 1) Pointwise convergence, Uniform convergence, almost everywhere convergence.
- 2) Lebesgue theorem, Egorov theorem, Luzin's theorem.
- 3) Square-summable and p-summable functions.
- 4) Holder's and Minkowski's inequality for functions and numbers.
- 5) Derived numbers and derivatives.
- 6) Increasing functions and functions of finite variation.
- 7) Cantor set and Cantor function.
- 8) Fourier series.

- 1. Theory of Functions of a real variable Volume-I by I. P. Natanson, Frederic Ungag Publishing Co., New York 1964.
- 2. Methods of Real Analysis by Richard Goldberg, Oxford & IBH Publishing Company, 1964.
- 3. Measure and Integration by Salish Shirali and H L Vasudeva, Springer, 2019.
- 4. Real Analysis by H. L. Royden and P M Fitzpatrick, Pearson Education Asia Ltd. and China Press, Fourth Edition, 2010.
- 5. Measure and Integration by I. K. Rana, Narosa Publishing House (1997).

## MAT412PR – Practical-II

#### (Based on MAT409: Complex Analysis-II and MAT410: Partial Differential Equations)

#### **Objectives and outcome:**

The objective here is to acquaint students to develop and consolidate their problem solving skills with the problems related to the notions that they have familiarize themselves in Complex Analysis-II and Partial Differential Equation courses. It is expected that on successful completion of this course the students will be able to handle problems related to the topics mentioned in the title with enhanced Mathematical Maturity.

#### Practicals Based on MAT409: Complex Analysis-II

- 1) Examples based on finding Laurent series, Taylor series. Division of Power Series.
- 2) Examples based on finding Residues; three types of isolated singular points. Examples Based on Cauchy's Residue theorem.
- 3) Examples Based on Residues at Infinity and its application (i.e. Theorem on Page 238 of the Textbook).
- 4) Examples on finding residues at poles i.e. Based on  $\phi$  Method. Examples based on zeroes, poles and special method for finding Residues at a simple Pole (i.e. Based on Theorem 2 on page 253).
- 5) Practical Based on finding improper integrals using Residue Theory.
- 6) Practical Based on finding integrals from Fourier Analysis and Based on Indented Path Technique.
- 7) Practical Based on finding definite integrals involving SINES and COSINES using residue theory.
- 8) Practical based on, Rouché's Theorem and Möbius Transformations (Bi-linear transformation). Miscellaneous Problems.

#### Based on MAT410: Partial Differential Equations

Practicals Based on

- 1) Lagrange's equation and Pfaffian Differential Equations
- 2) Charpit's Method and Jacobi's Method
- 3) Integrals Surfaces through a given curve
- 4) Cauchy Problem for Quasi Linear Equation and Non-Linear First Order PDE
- 5) Method of Separation of Variables
- 6) Various Initial and Boundary Value Problems for the Wave Equation
- 7) Various Initial and Boundary Value Problems for the Heat Equation
- 8) Various Initial and Boundary Value Problems for the Laplace Equation

#### Text book:

"An Elementary Course in Partial Differential Equations" (2<sup>nd</sup> Edition)

– T. Amaranath, Narosa PublishingHouse Pvt. Ltd., 2005.

- 1. Elements of Partial Differential Equations: Ian Sneddon, Mc-Graw-Hill International Editions
- 2. Ordinary and Partial Differential Equations: Theory and Application Nita H Shah, PHI Learning Pvt Ltd.
- 3. Partial Differential Equations: Methods, Applications and Theories Harumi Hattori, World Scientific
- 4. Elements of Partial Differential Equations(2<sup>nd</sup> Edition) Pavel Drabek, Gabriela Holubova, DE GRUYTER
- 5. Linear Partial Differential Equation for Scientists and Engineers: by Tyn Myint-U and Lokenath Debnath, Fourth Edition, Birkauser Publications
- 6. Methods of Mathematical Physics Vol.2, R. Courant and D. Hilbert, Wiley Eastern Pvt. Ltd., 1975.
- **Note:** Whenever possible it is expected that teachers will incorporate and encourage the use of packages like Mathematica/MATLAB/Maple/etc. in solving the problems in practical sessions.

## MAT501 – Functional analysis-I

#### **Objectives and Outcome:**

To introduce the elementary aspects of Banach Spaces and operators.

- **Unit I:** Review of linear spaces, quotient linear spaces, direct sums of linear subspaces, basis of a linear space existence using Zorn's lemma, linear transformations from a linear space to another, projections on a linear space.
- **Unit II:** Normed linear spaces, Banach spaces, quotient of a normed linear space by a closed linear subspace, continuous linear transformations from a normed linear space to a normed linear space, finite dimensional normed linear spaces.
- **Unit III:** Conjugate space of a normed linear space, Hahn-Banach theorem with consequences, the natural imbedding of a normed linear space in its second conjugate space, Reflexive spaces, open mapping theorem, projections on a Banach space, closed graph theorem.
- **Unit IV:** The uniform boundedness theorem, conjugate of an operator on a Banach space, Hilbert spaces, orthogonal complements, complete orthonormal sets in a Hilbert space.

#### Text book:

Introduction to topology and modern analysis by G. F. Simmons, McGraw - Hill Book Co.1963; Chapter 8 (42 onwards) to Ch.10 (upto 54).

- 1. "Functional analysis" by B. V. Limaye, New Age International Limited publishers.
- "An introduction to Hilbert Spaces" by S. K. Berberian, Oxford Uni. Press, 1959.
  D.Van Nostrand Co. Inc. Princeton, N.J., 1967, also available in paperback, edition by Springer.
- 3. "A Hilbert space problem book" by P. R. Halmos.
- 4. "An introductory functional analysis with application" by E.Kreyszig, WSE edition, 1989, paperback.
- 5. "Linear Analysis" An Introductory course by Bela Bollobas, Foundation Books, Delhi (Cheap edition), 1994.

## MAT502 – Algebra - I

#### **Objectives and Outcome:**

The objective of this course is to make students familiar with group structures and some related applications of the group. It also provides the basis for further studies.

- **Unit I:** Introduction to Groups, Symmetries of a Square, The Dihedral Groups, Elementary Properties of Groups, Finite Groups, Subgroups, Subgroup Tests, Cyclic Groups, Classification of Subgroups of Cyclic Groups.
- Unit II: Permutation Groups, Cycle Notation, Properties of Permutations, Isomorphisms, Cayley's Theorem, Properties of Isomorphisms, Automorphisms, Properties of Cosets, Lagrange's Theorem and consequences and further applications. External Direct Products and their properties. The Group of Units Modulo n as an External Direct product and further applications.
- **Unit III:** Normal subgroups, Factor groups, Applications of Factor groups. Internal Direct products, Group homomorphisms and their properties, the First Isomorphism Theorem. Fundamental theorem of finite Abelian groups, Isomorphism classes of Abelian groups.
- **Unit IV:** The Conjugacy classes, The Class Equation, Sylow theorems and their applications. Finite Simple groups, Nonsimplicity Tests, The simplicity of the group A<sub>5</sub>.

#### Text Book:

"Contemporary Abstract Algebra" (8<sup>th</sup> Edition) by Joseph A Gallian, Cengage Learning, 2013.

Chapters: 1 to 11; 24 and 25.

#### **References:**

- 1. "Basic Abstract Algebra" (2nd Edition) PB Bhattacharya, SK Jain, SR Nagpaul, Cambridge Uni Press, 1995.
- 2. "Algebra"-Michael Artin, PHI Learning Pvt Ltd, New Delhi.
- 3. "A Course in Algebra" EB Vinberg, American Mathematical Society, 2003.
- 4. "Algebra"- Thomas W Hungerford, Springer, 2004.

## MAT503EA – Advance Calculus

#### **Objectives:**

To do the theory and applications of Differential and Integral Calculus in several variables, and to gain facility with calculations.

- **Unit I:** Differentiability in several variables; the chain rule; the mean value theorem; higher order partial derivatives; Taylor's theorem; critical points; extreme value problems; Frechet derivatives.
- **Unit II:** The Implicit Function Theorem; curves and surfaces; transformations and coordinate systems; functional dependence.
- **Unit III:** Integration in higher dimensions; multiple integrals and iterated integrals; change of variables for multiple integrals; functions defined by integrals.
- **Unit IV:** Arc length and line integrals; Green's theorem; surface area and surface integrals; vector derivatives; the divergence theorem; some applications to physics; Stoke's theorem.

#### Text book:

Advanced Calculus by Gerald B. Folland, Pearson India Education Services Pvt. Ltd., 2012, Chapters 2-5.

- 1. Advanced Calculus by David V. Widder, Prentice Hall of India Pvt. Ltd., New Delhi, 1968.
- 2. Advanced Calculus by Patrick M. Fitzpatrick, The Sally Series. Indian Edition. AMS. ISBN 978-0-8218-5209.

## **MAT503EB – Financial Mathematics**

**Objectives and outcome:** The course aim to introduce the basic concepts and products of modern financial mathematics. Basic principles to understand and think about finance mathematically are introduced. The topics expose the user to fundamental concepts such as cash flows, present value, future value, yield and probability that form the basis for further advanced learning. The importance of the assumptions underlying the models leading to correct evaluation of risk are introduced and the central problem of finance i.e the quantification of risk and thereby the creation and pricing of contracts is introduced. It intends to provide the students an introduction to the theoretical foundations and main tools used in the pricing of bonds, European, American and Exotic Options, Futures and options on Futures.

**Basic Concepts (Prerequisites):** Arbitrage, Return and Investment, Time Value of Money, Bonds, Shares and Indices, Models and Assumptions

#### **UNIT I: Deterministic Cash Flows**

Net Present Value, Internal Rate of Return, Comparison of IRR and NPV, Bonds: Price and Yield, Clean and Dirty Price, Price-Yield curves, Duration, Term Structure of Interest rates, Immunisation, Convexity, Callable Bonds

#### **UNIT II: Random Cash Flows**

Random Returns, Portfolio Diagrams and Efficiency, Feasible Set, Markowitz Model, Capital Asset Pricing Model, Diversification, CAPM as a pricing formula, Numerical Techniques

### **UNIT III: Option Pricing for Discrete Time**

**Forwards and Futures:** Forwards and Futures, Price and Contract, Method of Replicating Portfolios, Hedging with futures, Currency futures, Stock Index Futures

**Option:** Option, Types of Option: Call & Put, European Option, American Option, Option Payoff

**Option Pricing for Discrete Time:** Binomial Option Pricing Model (1-Step, 2-Step, Many Step: Both American and European Option), Factors Influencing Option Premiums, Dynamic Hedging with Binomial Approach

#### **UNIT IV: Option Pricing for Continuous Time**

**Basic Concept for Black-Scholes:** Lognormal Distribution, Building Brownian Motion from a Random Walk, Geometric Brownian Motion, Risk-Neutral Valuation, Implied Volatility

**Derivation of Black-Scholes Formula:** The Black-Scholes Formula, Call-Put Parity, Options on Futures (Black-Scholes Approach), Dynamic Hedging with Black-Scholes Approach

**Derivation of Black-Scholes PDE:** Itô's Lemma & Derivation, The Black-Scholes PDE, Greeks

### References

- 1. Hull, J. C. Options, Futures and Other Financial Derivatives, Prentice Hall, 8th edition.
- 2. Amber Habib, The Calculus of Finance, Universities Press.
- 3. Capinski, Zastawniak, Mathematics for Finance: An Introduction to Financial Engineering, Springer.
- 4. Reilly & Brown, Investment Analysis and Portfolio Management, South-Western Publication.
- 5. Bodie, Kane & Marcus, Investment, McGraw-Hill.
- 6. J.B. Hunt and J.E. Kennedy, Financial Derivatives in Theory and Practice, Wiley, 2005.
- 7. M. Baxter and A. Rennie, Financial Calculus: An introduction to Derivative Pricing (Cambridge, UK), 1996.
- 8. R. J. Williams, Introduction to the Mathematics of Finance, AMS, 2011.
- 9. Victor Goodman & Joseph Stampfli, The Mathematics of Finance: Modelling and Hedging, AMS.

### **MAT504EA – Mathematical Programming**

#### **Objectives and Outcome:**

The objective of this course is to explore various mathematical programming algorithms to solve real life problems.

Prerequisites: Formulation of linear programming problem: Graphical Method, Simplex method, Transportation problem, Assignment problem.

- **Unit I**: Simplex method, Two-Phase Method, Big-M Method, Dual linear programming, Post Optimality Analysis, Integer programming, Gomory's cutting plane method, Branch and Bound method.
- Unit II: Definition of project, Job and events, Construction of project network, Dummy activities and events, Critical Path Method (CPM), Calculations of float and its interpretations, PERT, Crashing a Project, Network Models: Transshipment Problem, Shortest Path Problem, Maximal Flow Problem.
- **Unit III**: Non-linear programming, Lagrangian method, Kuhn-Tucker conditions, Quadratic programming, Wolfe's method.
- Unit IV: Introduction to Dynamic programming, Characteristic of Dynamic programming, Shortest Route Problem, Cargo loading Problem, Single Additive Constraint, Multiplicative Separable Return Function, Single Additive Constraint, Additive Separable Return Function, Single multiplicative Constraint, Additive Separable Return Function, Solution of Linear Programming Problem, Fractional programming.

#### Text book:

Syllabus is roughly covered by "OPERATIONS RESEARCH" by Nita H. Shah, Ravi M. Gor and Hardik Soni, PHI Publications, New Delhi, 2007.

- 1. OPERATIONS RESEARCH: AN INTRODUCTION (Eighth edition) by H. A. Taha, PHI Publications, New Delhi, 2006.
- 2. INTRODUCTION TO OPERATIONS RESEARCH by F. S. Hiller and G. J. Liberman, McGraw Hills Publication.

## **MAT504EB – Computational Mathematics**

#### **Objectives and outcome:**

To study the advanced computational methods.

- **UNIT I:** Computer Arithmetic: Floating point numbers and round off errors, Absolute and relative errors, Polynomial Interpolation: Hermite's interpolation formula with error analysis, Richardson interpolation, splines and spline interpolation, Aitken extrapolation.
- **UNIT II:** Numerical differentiation, Gaussian quadrature, Romberg integration, adaptive quadrature, Solution of system of Linear equations: Matrix inversion, Jordan's method, Escalator method and iterative method, The LU and Cholesky factorizations, Pivoting and constructing an algorithm Based on Gaussian elimination method.
- **UNIT III:** Algebraic Eigenvalue problem, Properties of Eigenvalues and eigen vectors, Power method, Inverse power method, Orthogonal factorization, QR algorithm for Eigenvalue problem.
- **UNIT IV:** Eigenvalues of complex matrix and complex eigen vectors, Numerical Solution of ODE: single step method-Runge-Kutta methods, Multistep method Milne Simpson's method, System of non-linear equations: Newton Raphson's method.

- 1. C. E. Froberg: Introduction to Numerical Analysis, Addision Wesley publishing Company, sixth edition, 1981.
- 2. Nita H Shah: Numerical Methods with C++ programming & Matlab, Second Edition, PHI.
- 3. S.S. Sastri: Introductory Methods of Numerical Analysis, Prentice Hall of India, New Delhi, 1997.
- 4. E.V. Krishnamurthy and S. K. Sen: Computer based numerical Algorithms, East–West press Pvt. Ltd. 1976.
- 5. Conte S. D and Carl deboor: Elementary Numerical Analysis: an algorithmic approach, Mc Graw Hill company, Third edition, 1981.
- 6. M. K. Jain: Numerical analysis for scientists and Engineers, New Age International Ltd. Publishing, 1992.
- 7. J. D. Faires and R. Burden: Numerical methods, second edition, Brooks/cole publishing Co., 1998.
- 8. E.Hairer, E. P. Norsett and G. Wanner: Solving ordinary differential equations I and II, Springer series in computational mathematics 8, Springer Berlin, 1993.

## MAT505PR – Practical I

#### Based on Functional Analysis - I:

- 1) Vector space and related concepts.
- 2) Linear transformations.
- 3) Examples of normed linear spaces and Banach spaces.
- 4) Continuous Linear transformations.
- 5) The conjugate and the second conjugate spaces of normed linear spaces.
- 6) The open mapping and the closed graph theorems; the uniform boundedness principle.
- 7) Hilbert space and related concepts.
- 8) Hilbert space and related concepts.

#### **Reference books:**

- 1. Introduction to topology and modern analysis by G. F. Simmons, McGraw Hill Book Co. 1963.
- 2. Functional analysis by B. V. Limaye, New Age International Limited publishers.

#### **Based on Algebra-I (Group Theory)**

- 1) Dihedral groups D<sub>n</sub>.
- 2) Properties of groups and subgroups.
- 3) Permutations groups, Isomorphisms.
- 4) Cosets and Lagrange's theorem, External direct products.
- 5) Normal subgroups and Factor groups.
- 6) Group homomorphisms, Isomorphism classes of finite Abelian group
- 7) Applications of Sylow theorems.
- 8) Finite simple groups.

- 1. "Basic Abstract Algebra" (2<sup>nd</sup> Edition) P B Bhattacharya, S K Jain, S R Nagpaul, Cambridge Uni Press, 1995.
- 2. "Algebra" Michael Artin, PHI Learning Pvt Ltd, New Delhi.
- 3. "A Course in Algebra" EB Vinberg, American Mathematical Society, 2003.
- 4. "Algebra" Thomas W Hungerford, Springer, 2004.
- **Note:** Whenever possible it is expected that teachers will incorporate and encourage the use of packages like Mathematica/MATLAB/Maple/etc. in solving the problems in practical sessions.

## MAT506PR – Practical II

#### Based on MAT503EA Advanced Calculus:

- Practical: 1-2 Differentiability in several variables;
- **Practical: 3-4** The implicit function theorem and its applications.
- **Practical: 5-6** Integration in higher dimensions;
- Practical: 7-8 Line and Surface Integrals.

#### Based on MAT503EB Financial Mathematics:

- 1) Problems based on concepts of Net Present Value, Internal Rate of Return and its comparison.
- 2) Problems based on concepts of Clean Price & Dirty Price of bond and Term Structure of Interest Rates.
- 3) Problems based on concepts of Risk and Return of a Portfolio, Markowitz Model and Efficient Frontier.
- 4) Problems based on concepts of Capital Asset Price Model, SML, CML and Sharp Ratio.
- 5) Problems based on concepts of payoff of option, Delta hedging and Replicating portfolio.
- 6) Problems based on concepts of Binomial Tree model (1-step, 2-step and n-step).
- 7) Problems based on concepts of Geometric Brownian Motion and Black-Scholes Model for option pricing.
- 8) Problems based on concepts of Put-Call Parity and Greeks.

#### Text Book:

The course is covered by Chapters 1-8 of The Calculus of Finance: by Amber Habib, Universities Press

#### **References:**

- 1. Hull, J. C. Options, Futures and Other Financial Derivatives, , Prentice Hall, 8th edition.
- 2. Pascucci, A. Bocconi, PDE and Martingale Methods in Option Pricing. & Springer Series, 2011.
- 3. R. U. Seydel, Tools for Computational Finance, University text, 3rd Edition Springer 2000.
- 4. J.B. Hunt and J.E. Kennedy, Financial Derivatives in Theory and Practice, Wiley, 2005.
- 5. M. Baxter and A. Rennie, Financial Calculus: An introduction to Derivative Pricing (Cambridge, UK), 1996.

[Cont.]

#### **Based on MAT504EA Mathematical Programming**

#### **Objectives and Outcome:**

The objective of this paper is to apply techniques studied in MAT504EA to solve problems.

- 1) Formulation of LPP.
- 2) Solve problem using Simplex method.
- 3) Solve problems using Big M method and Two-phase method.
- 4) Solve transportation problem using North-West Corner method, Least cost method and Vogel's method.
- 5) Solve transportation problems using MODI's method.
- 6) Solve LPP for integer requirement.
- 7) Obtain quadratic form of NLPP. Solve NLPP using KT conditions, Lagrange's multiplier, Wolfe method and Beales method.
- 8) Solve NLPP using dynamic programming algorithm and fractional programming.

#### **Based on MAT504EB Computational Mathematics**

- 1. Polynomial Interpolation: Hermite's interpolation formula with error analysis.
- 2. Richardson interpolation, Splines and spline interpolation, Aitken extrapolation.
- 3. Jordan's method.
- 4. The LU and Cholesky factorizations.
- 5. Pivoting and constructing an algorithm Based on Gaussian elimination method.
- 6. Algebraic Eigenvalue problem.
- 7. Orthogonal factorization, QR algorithm for Eigenvalue problem.
- 8. Numerical Solution of ODE.

- 1. C. E. Froberg: Introduction to Numerical Analysis, Addision Wesley publishing Company, sixth edition, 1981.
- 2. Nita H Shah: Numerical Methods with C++ programming & Matlab, Second Edition, PHI
- 3. S. S. Sastri: Introductory Methods of Numerical Analysis, Prentice Hall of India, New Delhi, 1997.
- 4. E. V. Krishnamurthy and S.K.Sen: Computer based numerical Algorithms, East-West press Pvt. Ltd. 1976
- 5. Conte S. D and Carl deboor: Elementary Numerical Analysis: an algorithmic approach, Mc Graw Hill company, Third edition, 1981
- 6. M.K. Jain: Numerical analysis for scientists and Engineers, New Age International Ltd. Publishing, 1992
- 7. J. D. Faires and R. Burden: Numerical methods, second edition, Brooks/cole publishing Co., 1998.
- 8. E. Hairer, E. P. Norsett and G. Wanner: Solving ordinary differential equations I and II, Springer series in computational mathematics 8, Springer Berlin, 1993.
- **Note:** Whenever possible it is expected that teachers will incorporate and encourage the use of packages like Mathematica/MATLAB/Maple/etc. in solving the problems in practical sessions.

## **MAT507 – Differential Geometry**

#### **Objectives:**

To study curves and surfaces. The approach is straight-forward, and may help in teaching geometry in schools.

- **Unit I:** Concept of a curve; tangent to a curve; osculating planes; envelope of a family of plane curves; natural parametrization; curvature; torsion; Frenet formulas.
- **Unit II:** Concept of a surface; tangent planes; osculating paraboloid; classification of surface points.
- **Unit III:** Area of a surface; normal curvature; lines of curvature; mean and Gaussian curvature; a surface of constant negative Gaussian curvature.
- **Unit IV:** Intrinsic geometry of surfaces; Gaussian curvature is intrinsic; geodesic lines; extremal property of geodesics; Gauss-Bonnet theorem; closed surfaces.

#### Text book:

Geometry by A. Pogorelov, Mir Publishers, 1987. Chapters VIII – XII.

- 1. Elementary Differential Geometry, by Andrew Pressley, Spinger 2001.
- 2. Differential Geometry of Curves and Surfaces; by Manfredo P. do Carmo. Prentice-Hall Inc., New Jersey.
- 3. Differential Geometry, by A. V. Pogorelov; (Publisher) P. Noordhoff N. V. Groningen The Netherlands.

## MAT508 – Algebra II

### **Objectives:**

To study ring theory further, and to introduce Galois theory with some applications.

- **Unit I:** A brief review; ideals and factor rings; ring homomorphisms.
- **Unit II:** Polynomial rings; irreducibility tests; unique factorization in Z[X]; divisibility in integral domains.
- **Unit III:** Splitting fields; finite extensions of fields; properties of algebras extensions; finite fields; geometric constructions.
- **Unit IV:** Fundamental theorem of Galois theory; solvability of polynomials by radicals; insolvability of a quantic; cyclotomic polynomials.

#### Text book:

Contemporary Abstract Algebra (Eighth Edition) by Joseph A. Gallian, Cengage India Learning Private Limited, 2013. Chapters: 12-23, 32,33

- 1. A Course in Algebra by E. B. Vinberg, American Mathematical Society, 2003.
- Basic Abstract Algebra (Second Edition) by P. B. Bhattacharya, S. K. Jain, S. R. Nagpal, Cambridge University press, 1995.

## **MAT509EA – Number Theory**

#### **Objectives and outcome:**

The main objective is to give a simple account of elementary classical number theory.

- **Unit I:** Review of Divisibility, Arithmetical Functions: The function [x], multiplicative functions, Euler's (totient) function  $\varphi(n)$ , The Mobius function  $\mu(n)$ , The functions  $\tau(n)$  and  $\sigma(n)$ , Average orders, Perfect numbers.
- **Unit II:** Congruences: Definitions, Chinese-Remainder theorem, The theorems of Fermat and Euler, Wilson's theorem, Lagrange's theorem, Primitive roots, indices.
- **Unit III:** Quadratic residues, Legendre's symbol, Euler criterion, Gauss's lemma, Law of quadratic reciprocity, Jacobi's symbol.
- **Unit IV:** Dirichlet theorem, Continued Fractions, Rational approximations, Quadratic irrationals, linear Diophantine equations ax + by = c, Pell's equations, Pythagorean triples,

#### Text book:

A Concise introduction to the Theory of Numbers by Alan Baker, Cambridge Uni. Press, Cambridge.

- 1. "An introduction to the Theory of Numbers (5th edition) by Ivan Nivan, H. S. Zuckerman, H. L. Montgomery, John Wiley & Sons Inc., 2000.
- 2. Elementary Number theory (6<sup>th</sup> Edition) by David M. Burton, Tata McGraw-Hill, 2007.

## MAT509EB – Mathematical Methods

#### **Objectives and Outcome:**

The objective of this course is to explore various numerical techniques to solve real life problems.

- **Unit I:** Laplace Transform. Linearity. First Shifting Theorem (s-Shifting), Transforms of Derivatives and Integrals. ODEs, Unit Step Function (Heaviside Function), Second Shifting Theorem (t-Shifting), Short Impulses. Dirac's Delta Function. Partial Fractions, Convolution, Integral Equations, Differentiation and Integration of Transforms., ODEs with Variable Coefficients, Systems of ODEs
- Unit II: Fourier Analysis: Fourier Series, Arbitrary Period. Even and Odd Functions. Half-Range Expansions, Approximation by Trigonometric Polynomials, Sturm–Liouville Problems. Orthogonal Functions, Orthogonal Series. Generalized Fourier Series, Fourier Integral, Fourier Cosine and Sine Transforms, Fourier Transform. Discrete and Fast Fourier Transforms
- **Unit III**: Introduction and properties of Z-transform, change of scale and shifting property. Inverse Z-transforms, Multiplication and division by K, Initial and Final value, Partial sums, Convolution property of Casual sequence, Inverse of Z-transform by division, binomial expansion and partial fractions, Inversion by Residue Method, Solution of difference equations.
- **Unit IV**: Introduction and properties of Hankel transform, Inversion formula, Parseval's theorem, Henkel's transform of the derivative of the function, Application to boundary problems, Finite Hankel transmission.

#### **Text Books:**

For Unit I, II: Advanced Engineering Mathematics, Erwin Kreyszig, Wiley Student Edition, Tenth Edition 2011.

For Unit III, IV: Advanced Engineering Mathematics, H. K. Dass, S. Chand 2006.

- 1. Advanced Engineering Mathematics, (10<sup>th</sup> Edition) Erwin Kreyszig, John Wiley and Sons, Inc., 2011.
- 2. Differential Equations and their Applications (2<sup>nd</sup> Edition), M. Brann, Sringer Verlag, New York Inc, 1978.
- 3. Advanced Engineering Mathematics, H. K. Dass, S. Chand 2006.
- 4. Mathematical methods for Physicists 6<sup>th</sup> edition by Arfken & Weber (Academic press Indian Reprint).
- 5. Methods of Mathematical Physics (Vol. I & II) by Courant & Hilbert (1953) New York Interscience.
- 6. Integral Transforms and their Applications (3<sup>rd</sup> Edition), Lokenath Debnath and Dambaru Bhatta, CRC Press, 2015.

## **MAT510EA – Quantitative Techniques**

#### **Objectives and Outcome:**

The objective of this course is to explore various techniques to solve real life problems.

- **Unit I:** Inventory Models: Concept, classification of inventory models, EOQ model, EPQ model, EOQ model with shortages, EOQ model with constraints, Probabilistic Inventory Models: News Vendor Problem & S-s Model.
- Unit II: Queuing Models: Introduction to Queues, Pure Birth and Death Model, Operating Characteristics of a Queueing System, Queueing models: ((M/M/1): (∞/FCFS)), ((M/M/1): (N/FCFS)), ((M/M/c): (∞/FCFS)), Economic Analysis of Queueing Models.
- **Unit III:** Game Theory: Characteristics of a Game, Maximin and Minimax principle, Saddle point, Two-person Zero-sum games with saddle point, Game problems without saddle point, Pure strategy and mixed strategy, Dominance Rule, Solution of a game problem without saddle point: Graphical method for 2×n & n×2 game, Algebraic method, Linear Programing Method.
- **Unit IV:** Decision Analysis: Introduction, Decision under certainty, Decision under risk, Decision under uncertainty, Decision tree, Calculation of posterior probabilities by Bayes Rule and calculation of EPPI, EVPI and EVSI.

Simulation and its applications in solving queueing and inventory problems.

#### **Text Book:**

Syllabus is roughly covered by "OPERATIONS RESEARCH" by Nita H. Shah, Ravi M. Gor and Hardik Soni, PHI Publications, New Delhi, 2007.

- 1. ANALYSIS OF INVENTORY SYSYTEMS by G. Hadley and T. M. Whitin, Prentice Hall.
- 2. INVENTORY SYSTEMS by E. Naddor, John Wiley Publications, 1964.
- 3. SIMULATION by Gordan, Prentice Hall.

## MAT510EB – Functional Analysis II

#### **Objectives and outcome:**

To analyse the concepts related to Hilbert spaces and Banach spaces.

- **Unit I:** Conjugate space of a Hilbert space, Adjoint of an operator on a Hilbert space, Selfadjoint operators, Normal & unitary operators.
- **Unit II:** Projections on a Hilbert space, Invariant subspaces. Spectral resolution of an operator on a finite dimensional Hilbert space. The spectral theorem for finite dimensional Hilbert spaces and its consequences.
- **Unit III:** Classification of the spectrum of an operator on a Banach space, Spectrum of the Shift operator, Multiplication operator etc. Gelfand Mazur theorem, The Spectral radius formula.
- **Unit IV:** Compact operators. The Spectrum of compact operators.

#### Text Books:

 "Introduction to Topology and Modern analysis" by G. F. Simmons, McGraw – Hill Book Co. 1963.

Chapter 10 (55 onwards), Chapter 11 (61 to 63).

 "Functional analysis" by B. V. Limaye, New Age International Limited publishers, 1996. (Second Edition). Chapter 3(Article 12), Chapter 5(Article 17, 18).

#### **References:**

- 1. "An introduction to Hilbert Spaces" by S. K. Berberian, Oxford Uni. Press, 1959, D. Van Nostrand Co. Inc. Princeton, N.J., 1967, also available in paperback, edition by Springer.
- 2. "A Hilbert space problem book" by P. R. Halmos.
- 3. "An Introductory Functional Analysis with application" by E. Kreyszig, WSE edition, 1989, paperback.

## MAT511PR – Practical

## **Problem-Solving**

#### Unit I: Abstract Algebra

Groups, subgroups, normal subgroups, quotient groups, homeomorphisms, cyclic groups, permutation groups, Cayley's theorem, class equations, Sylow theorems. Rings, ideals, prime and maximal ideals, quotient rings, Polynomial rings and irreducibility criteria.

#### Unit II: Real Analysis

Elementary set theory, finite, countable and uncountable sets. Real number system as a complete ordered field, Archimedean property, supremum, infimum. Sequences and series, convergence, limsup, liminf. Bolzano Weierstrass theorem, Heine Borel theorem. Continuity, uniform continuity, differentiability, mean value theorem. Metric spaces, compactness, connectedness.

#### Unit III: Linear Algebra

Vector spaces, subspaces, linear dependence, basis, dimension, algebra of linear transformations. Algebra of matrices, rank and determinant of matrices. Eigenvalues and eigenvectors, Cayley-Hamilton theorem. Matrix representation of linear transformations. Change of basis, canonical forms, diagonal forms, triangular forms, Jordan forms. Inner product spaces, orthonormal basis. Quadratic forms, reduction and classification of quadratic forms.

#### **Unit IV: Complex Analysis**

Algebra of complex numbers, the complex plane, polynomials, power series, transcendental functions such as exponential, trigonometric and hyperbolic functions. Analytic functions, Cauchy-Riemann equations. Contour integral, Cauchy's theorem, Cauchy's integral formula, Liouville's theorem, Maximum modulus principle, Schwarz lemma, Open mapping theorem. Taylor series, Laurent series, calculus of residues.

#### **Reference Books:**

- 1. J. A. Gallian, (2012), Contemporary Abstract Algebra (8<sup>th</sup> Edition), Brooks/Cole Pub Co.
- 2. R. G. Bartle, D. R. Sherbert, (2014), Introduction to Real Analysis (4<sup>th</sup> Edition), Wiley Publication.
- 3. K. Hoffman and R. Kunze, (2018), Linear Algebra (2<sup>nd</sup> Edition), Prentice Hall India Learning Private Limited.
- 4. J. W. Brown, R. V. Churchill, (2021), Complex Variables and Applications (9<sup>th</sup> Edition), McGraw Hill Education (India) Private Limited

**Note:** Whenever possible it is expected that teachers will incorporate and encourage the use of packages like Mathematica/MATLAB/Maple/etc. in solving the problems in practical sessions.

## **MAT512PR Project**

### **Objectives and Outcome:**

Through project, students will explore real-world problems and challenges, simultaneously developing cross-curriculum skills. This type of learning is filled with active and engaged learning, it inspires students to obtain a deeper knowledge of the subjects they're studying. The student will attain the knowledge gained through this approach far more readily than through traditional textbook-centered learning. In addition, students develop confidence and self-direction as they do independent work.